Séminaire de Probabilités et Statistiques

Vendredi 9 décembre à 14h00
Laboratoire Dieudonné
Salle de Conférences

Alexandre Tsybakov
(CREST-ENSAE)

Slope meets Lasso : improved oracle bounds and optimality

We show that two polynomial time methods, a Lasso estimator with adaptively chosen tuning parameter and a Slope estimator, adaptively achieve the exact minimax prediction and ℓ_2 estimation rate $(s/n)\log(p/s)$ in high-dimensional linear regression on the class of s-sparse target vectors in $\mathbf{R}^p$. This is done under the Restricted Eigenvalue (RE) condition for the Lasso and under a slightly more constraining assumption on the design for the Slope. The main results have the form of sharp oracle inequalities accounting for the model misspecification error. The minimax optimal bounds are also obtained for the ℓ_q estimation errors with $1 \leq q \leq 2$ when the model is well-specified. The results are non-asymptotic, and hold both in probability and in expectation. The assumptions that we impose on the design are satisfied with high probability for a large class of random matrices with independent and possibly anisotropically distributed rows. We give a comparative analysis of conditions, under which oracle bounds for the Lasso and Slope estimators can be obtained. In particular, we show that several known conditions, such as the RE condition and the sparse eigenvalue condition are equivalent if the ℓ_2-norms of regressors are uniformly bounded. This is a joint work with Pierre C. Bellec and Guillaume Lecué.