Learning Dynamic Utility from Monotonic Characteristic Processes.

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Abstract

Decision making under uncertainty is generally considered as the selection of an optimal sequence of actions in an uncertain environment. Its calibration raises the "inverse" problem to recover the criterium from the data. A classical example in economy is the theory of "revealed preference" introduced by Samuelson in the 40's, [?]. The observable at a given date t (discrete, or real) is the increasing (in x) characteristic process $\mathcal{X}_t(x)$. The objective is to recover a concave dynamic stochastic utility $\{U(t,z)\}$, "revealed" in the sense where its performance is without bias, more formally when "$\{U(t,\mathcal{X}_t(x))\}$ is a martingale".
Given the concavity of the criterium, tools of convex analysis play a large role, in particular the duality between the concave utility $U(t,z)$ and its Fenchel convex transform $\widetilde{U}(t,y)$ via the marginal utility $U_z(t,z)$. Considering together the primal and dual problems to recover $(\mathbf{U},\widetilde{\mathbf{U}})$ from two observable characteristic processes $(\mathcal{X}_t(x),Y_t(y))$ yields to the necessarily condition that $(\mathcal{X}_t(x)Y^c_t(u_x(x)))$ is a martingale and by means of a simple stochastic change of variable that the marginal utility $\mathbf{U}_z$ is given by $U_z(t,z)=Y^c_t(u_x(X^{c,-1}_t(z))$.
We can go further by focusing on the $(u,\mathcal{X},Y)$ triplets, to obtain a quasi necessary et sufficient condition for the existence of a solution : "$\{\mathcal{X}_t(x)Y_t(y)\}$ is a supermartingale for any (x,y) and $\{\mathcal{X}_t(x)Y_t(u_z(x))\}$ is a martingale for $y=u_z(x)$ ". Moreover, there is an equivalent intrinsic framework, where in addition the processes "$\{\mathcal{X}_t(x)\}$,$\{Y_t(y)\}$,$\{U(t,z)\}$" are supermartingales.
Itô's semimartingale framework is used only to illustrate this characterization. The operational version ensures that the revealed utility is solution of a non-linear SPDE. Less obvious is its interpretation as stochastic value function of some optimization problem. Financial markets framework appears as a special case, under stronger assumptions. Then, we revisit the dynamic equilibrium problem as in He and Leland, by considering it as a revealed utility problem. We solve the problem in random environment, by characterizing all the equilibria, in showing that the only possible conjugate dynamic utilities are the mixture of stochastic dual power utilities.