Séminaire de Probabilités et Statistique

Mardi 4 janvier à 14h00

Salle Fizeau (5ème étage)

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Flocking of complex roots of random polynomials, under iterated differentiation

I will first describe the unexpected "pairing" between polynomial roots and critical points and then, the induced "collective motion" of roots sets of high degrees random polynomials P_n, under iterated differentiation, i.e. ${P_n}' = P_{n-1}$. The asymptotic motion, when n tends to infinity, has been recently studied in the 1D case, i.e. where all the roots are real or have a rotational symmetry. This is also related to some topics in Random matrices theory. A non local PDE has been derived by Sternerberger to rule this 1D motion. We propose to generalise it in 2D, for a large class of complex roots sets. We are also strongly interested by the case of random polynomials with real coefficients which have both conjugated complex roots and real roots; in that situation, the trajectories of the complex roots (under iterated differentiation) are "attracted" by the real axis. We will assume that, asymptotically, the empirical distributions of the roots sets tend to regular limit distributions, that we represent by their 2D cumulative functions $F(x, y)$. We then use a second order Taylor expansion of $F(x, y)$ to capture local stochastic geometric features, needed to compute the expressions of the proposed equations of motion.
This is a work in progress, a collaboration with Didier Clamond.