Séminaire de Probabilités et Statistique

Mardi 6 février à 14h00

Salle de conférences

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*How far is a tensor from being rank-*1*?*

The best rank-1 approximation ratio of a space T of tensors measures the largest angle/distance between a tensor t in T and its best rank-1 approximation. It is defined as the minimum of the ratio $R(t)$ of 2 norms on T and has appearances in several branches of mathematics. Thus, in quantum information theory, $R(t)$ is used to quantify entanglement of multipartite quantum states. Also, it is known that multiplication tensors of the 4 classical division algebras (reals, complex numbers, quaternions and octonions) minimize $R(t)$. In subspaces of symmetric tensors, the minimum value of $R(t)$ is linked to the existence of uniformly bounded sequences of homogeneous polynomials, a problem that goes back to Bourgain. With Agrachev and Uschmajew we showed that all minimizers of $R(t)$ on the space of binary symmetric tensors are obtained from Chebyshev polynomials of the first kind.
The talk will be focused on my joint work with Tonelli-Cueto, where we bound the minimum of R using probabilistic techniques (tail estimates of subgaussian random variables). In vague terms, our results imply that symmetric tensors are as far from being rank-1 as general ones.